

COMPARATIVE ANALYSIS OF METHODS OF RESERVOIR PERFORMANCE PREDICTION ON XY FIELD

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Abstract: To accurately estimate oil and gas reserves, plan for the development of the field, it requires a sound knowledge of engineers to adequately predict the performance of the reservoir since nobody can enter the reservoir to actually tell its content and behavior. It is therefore imperative to note that whatever result obtained from such prediction, must be within reasonable engineering tolerance of error. This implies that its success will depend solely on the accurate description of the reservoir rock properties, fluid properties, rock-fluid properties and flow performance. In the study, Tarner's, Tracy's and Schilthuis methods of reservoir performance prediction using material balance techniques was adopted to predict the performance of XY reservoir. Hence, after performing several iterations, the results obtained with Tracy's and Schilthuis' methods indicate a closeness with the XY reservoir performance prediction at 2870 psia, which is an indication that either Tracy's or Schilthuis method can be used to predict the XY reservoir performance at lower pressures as the reservoir declines.

Keywords: performance predictions, oil production, gas production, gas-oil ratio, reserves, material balance, rock properties, fluid properties.

I. INTRODUCTION

Prediction of reservoir performance is a vital aspect of the oil and gas business which guides management's decision of how the reservoir will behave in the future. This implies that its success will depend solely on the accurate description of the reservoir rock properties, fluid properties, rock-fluid properties and flow performance. Reservoir characterization is a process that continues in the entire life of the field, it helps to reduce or identify subsurface uncertainties associated with the static and dynamic reservoir model. A detailed reservoir rock characterization is not considered in material balance prediction methods but can be seen in reservoir simulation¹. In this study, the prediction methods used, basically incorporated the material balance equation (MBE) which is predominantly pressure-temperature-volume (PVT) properties.

Therefore, an appropriate description of reservoir fluid properties is also key. To determine these properties, the ideal process is to sample the reservoir fluid and perform laboratory studies on the fluid samples. This is not always possible to continuously take a fluid sample for analysis as the reservoir pressure declines, hence, engineers have resorted to correlations such as Petrosky and Fashad²,

II. HISTORY MATCHING

A model that cannot tell the past or previous performance of a reservoir within a reasonable tolerance of error is not good to predict the future performance, thus, the update of a model to fit the actual performance is known as history matching. To history match a given field data with material balance equation, Table 1 shows the known parameters to match and the unknown parameters to tune in order to get field historical production data.

Table 1: History match and prediction parameters

Known Parameters		
History Matching	Parameter	Symbol
	Production data	Np, Gp, Wp and Rp
	Hydrocarbon Properties	Boi, Bo, Bg, Bgi, Rsi, Rs
	Reservoir Properties	Sw, cw, cf, m
	Pressure drop	ΔP
Unknown Parameters		
Prediction	Reserves	N
	Water Influx	We
	Reserves, Water influx, Hydrocarbon properties, Reservoir properties	

It, therefore, implies that history matching is a process of adjusting key properties of the reservoir model to fit or match the actual historic data. It helps to identify the weaknesses in the available data, improves the reservoir description and forms basis for the future performance predictions. One of these parameters that are vital in history matching is the aquifer parameters which are not always known. Hence, modification of one or several of these parameters to obtain an acceptable match within reasonable engineering tolerance of error or engineering accuracy is history matching³.

III. ASSUMPTIONS AND LIMITATIONS OF MBE

There are several assumptions made by the engineers to successful carry out an evaluation on hydrocarbon reservoirs. These are:

- The reservoir is considered to be a tank
- Pressure, temperature, and rock and fluid properties are not space dependent
- Uniform hydrocarbon saturation and pressure distribution (homogenous reservoir)
- Thermodynamic equilibrium always attained.
- Isothermal condition apply
- Production data is reliable

The implication of these assumptions in evaluating reservoir performance is that, material balance uses a model that is existing as an imagination of the reservoir to actually tell or forecast the behavior of the reservoir, established as a result of the production of hydrocarbons from the reservoir with natural means or by gas or water injection.

- It is considered to be a tank model with a zero dimension which implies that it does not reflect the area drained
- the shape or geometry of the reservoir
- the manner in which the wells drilled into the reservoir are positioned and orientations are not considered
- the dynamic effects of fluid are not considered
- the heterogeneous nature of the reservoir and no time parameters

These implication lead to the statement made by, that the material balance method has some limitations, though it can be used as a pre-processing tool to infer fluid in place, drive mechanisms and identify aquifer for a more sophisticated tool “reservoir simulation” which gives insight into dynamic rock and fluid properties for evaluation of past reservoir performance, prediction of future reservoir performance, and reserves estimation.

IV. REVIEW OF PREDICTION METHODS

Reservoir performance prediction is an iterative process which requires that a convergence criterion must be met after a satisfactory history match is achieved to evaluate in a short period of time for a proper optimization of future reservoir management planning of a field. There are basically four methods of reservoir performance prediction applying material

balance concept and not a numerical approach where the reservoir is divided into grid blocks. These are:

- ✚ Tracy method
- ✚ Muskat method
- ✚ Tarner method
- ✚ Schilthuis method

All the techniques used to predict the future performance of a reservoir are based on combining the appropriate MBE with the instantaneous GOR using the proper saturation equation. The calculations are repeated at a series of assumed reservoir pressure drops. These calculations are usually based on one stock-tank barrel of oil-in-place at the bubble-point pressure. Above the bubble point pressure, the cumulative oil produced is calculated directly from the material balance equations which are expressed below.

V. TRACY PREDICTION METHOD

Tracy⁴ developed a model for reservoir performance prediction that did not consider oil reservoirs above the bubble-point pressure (undersaturated reservoir) but the computation starts at pressures below or at the bubble-point pressure. To use this method for predicting future performance, it is pertinent therefore to select future pressures at desired performance. This means that we need to select the pressure step to be used. Hence, Tracy's calculations are performed in series of pressure drops that proceed from a known reservoir condition at the previous reservoir pressure (P_{i-1}) to the new assumed lower pressure (P_i). The calculated results at the new reservoir pressure become "known" at the next assumed lower pressure. The cumulative gas, oil, and producing gas-oil ratio will be calculated at each selected pressure, so the goal is to determine a table of N_p , G_p , and R_p versus future reservoir static pressure.

Tracy's Prediction Algorithm

Step 1: Select an average reservoir pressure (P_i) of interest

Step 2: Calculate the values of the PVT functions ϕ_o , ϕ_g & ϕ_w where

$$\phi_o = \frac{B_o - R_s B_g}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right]}$$

$$\phi_g = \frac{B_g}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right]}$$

$$\phi_w = \frac{1}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right]}$$

Step 3: Assume (estimate) the GOR (R_i) at the pressure of interest

Step 4: Estimate the average instantaneous GOR (R_{avg}) at the pressure of interest

The average producing gas-oil ratio for a pressure decrement from P_{i-1} to the pressure of interest P_i given as:

$$R_{avg} = \frac{R_{i-1} + R_i}{2}$$

Step 5: Calculate the incremental cumulative oil production ΔN_p as:

The general material balance equation is given as

$$N = N_p \phi_o + G_p \phi_g - (W_e - W_p) \phi_w$$

For a solution gas drive reservoir (undersaturated reservoir) the equation reduces to

$$N = N_p \phi_o + G_p \phi_g$$

At pressure of interest

$$N = (N_p)_i (\phi_o)_i + (G_p)_i (\phi_g)_i$$

Note that as the pressure decreases, there is a corresponding incremental production of oil and gas designated as ΔN_p & ΔG_p . There the cumulative oil and gas production at pressure of interest are given as:

$$(N_p)_i = (N_p)_{i-1} + \Delta N_p$$

$$(G_p)_i = (G_p)_{i-1} + \Delta G_p$$

Substitute into the above equation of N at pressure of interest, we have

$$N = [(N_p)_{i-1} + \Delta N_p] (\phi_o)_i + [(G_p)_{i-1} + \Delta G_p] (\phi_g)_i$$

But

$$\Delta G_p = R_{avg} \Delta N_p$$

Hence

$$\begin{aligned} N &= [(N_p)_{i-1} + \Delta N_p] (\phi_o)_i + [(G_p)_{i-1} + R_{avg} \Delta N_p] (\phi_g)_i \\ N &= (N_p)_{i-1} (\phi_o)_i + \Delta N_p (\phi_o)_i + (G_p)_{i-1} (\phi_g)_i + R_{avg} \Delta N_p (\phi_g)_i \\ N - (N_p)_{i-1} (\phi_o)_i - (G_p)_{i-1} (\phi_g)_i &= \Delta N_p [(\phi_o)_i + R_{avg} (\phi_g)_i] \\ \therefore \Delta N_p &= \frac{N - (N_p)_{i-1} (\phi_o)_i - (G_p)_{i-1} (\phi_g)_i}{(\phi_o)_i + R_{avg} (\phi_g)_i} \end{aligned}$$

Step 6: Calculate total or cumulative oil production from

$$(N_p)_i = (N_p)_{i-1} + \Delta N_p$$

Step 7: Calculate the oil and gas saturations at pressure P_i when the cumulative oil production $(N_p)_i$ is given as (see derivation in Chapter 5):

$$\begin{aligned} S_o &= \left[1 - \frac{(N_p)_i}{N} \right] \left[\frac{(B_o)_i}{(B_{oi})} \right] [1 - S_{wi}] \\ S_g &= 1 - S_o - S_{wi} \end{aligned}$$

Step 8: Obtain the relative permeability ratio k_{rg}/k_{ro} at time i as a function of S_o or S_g or $S_L = (S_o + S_{wi})$.

Step 9: Make a plot of $\frac{k_{rg}}{k_{ro}}$ Versus S_o or S_L on a semi log graph

Step 10: Calculate the new instantaneous GOR at time, i given as

$$R_{avg}^{New} = \left[\frac{B_o k_{rg} \mu_o}{B_g k_{ro} \mu_g} \right]_i + (R_s)_i$$

Step 11: Compare the assumed or estimated GOR in Step 3 with the calculated GOR in Step 10. If the values are within acceptable tolerance, the incremental cumulative oil produced is correct (step 5), then proceed to the next step. If not within the tolerance, set the assumed GOR equal to the calculated new GOR and repeat the calculations from Step 3.

Step 12: Calculate the cumulative gas production.

$$(G_p)_i = (G_p)_{i-1} + \Delta G_p = (G_p)_{i-1} + R_{avg} \Delta N_p$$

Step 13. Make a final check on the accuracy of the prediction which should be made on the MBE as:

$$N_p \phi_o + G_p \phi_g = N \pm \textit{Tolerance}$$

If the STOIPP is based on 1 STB in step 5, the final check equation reduces to

$$N_p \phi_o + G_p \phi_g = 1 \pm \textit{Tolerance}$$

Step 14. Repeat from Step 1 for a new (lower) pressure value.

As the calculation progresses, a plot of GOR versus pressure can be maintained and extrapolated as an aid in estimating GOR at each new pressure.

VI. TARNER'S PREDICTION METHOD

Turner⁵ suggested an iterative technique for predicting cumulative oil production N_p and cumulative gas production G_p as a function of reservoir pressure. The method is based on solving the MBE and the instantaneous GOR equation simultaneously for a given reservoir pressure drop from a known pressure P_{i-1} to an assumed (new) pressure P_i . It is accordingly assumed that the cumulative oil and gas production has increased from known values of $(N_p)_{i-1}$ and $(G_p)_{i-1}$ at reservoir pressure P_{i-1} to future values of $(N_p)_i$ and $(G_p)_i$ at the assumed pressure P_i . To simplify the description of the proposed iterative procedure, the stepwise calculation is illustrated for a volumetric saturated oil reservoir; however, the method can be used to predict the volumetric behavior of reservoirs under different driving mechanisms.

Turner's method was preferred over Tracy and Muskat because of the differential form of expressing each parameter of the material balance equation by Tracy. Also, Turner and Muskat method using iterative approach in the prediction until a convergence is reached.

Furthermore, a first approach of the Cumulative Oil Production is needed before the calculation itself could be performed; a second value of this variable is calculated through the equation which defines the Cumulative Gas Production, as an average of two different moments in the production life of the reservoir; this expression, as we will see, is a function of the Instantaneous Gas Oil Rate, then we need also to calculate this value in advance from an equation derived from Darcy's law, this is a very important relationship since it is strongly affected by the relative permeability ratio between oil and gas. Finally, both values are compared, if the difference is within certain predefined tolerance, our first estimate of the Cumulative Oil Production will be considered essentially right, otherwise the entire process is repeated until the desired level of accuracy is reached (Turner⁷).

Turner's Prediction Algorithm

Step 1: Select a future reservoir pressure P_i below the initial (current) reservoir pressure P_{i-1} and obtain the necessary PVT data. Assume that the cumulative oil production has increased from $(N_p)_{i-1}$ to $(N_p)_i$. It should be pointed out that $(N_p)_{i-1}$ and $(G_p)_{i-1}$ are set equal to zero at the bubble-point pressure (initial reservoir pressure).

Step 2: Estimate or guess the cumulative oil production $(N_p)_i$ at P_i .

Step 3: Calculate the cumulative gas production $(G_p)_i$ by rearranging the MBE to give:

$$(G_p)_{MBE,i} = N \left(\{(R_{si})_{i-1} - (R_s)_i\} - \left\{ \frac{(B_{oi})_{i-1} - (B_o)_i}{(B_g)_i} \right\} \right) - (N_p)_i \left\{ \frac{B_o}{B_g} - R_s \right\}_i$$

Step 4: Calculate the oil and gas saturations $\{(S_o)_i \text{ and } (S_g)_i\}$ at the assumed cumulative oil production $(N_p)_i$ and the selected reservoir pressure P_i by applying Equations

$$(S_o)_i = (1 - S_{wi}) \left[1 - \frac{(N_p)_i}{N} \right] \left(\frac{(B_o)_i}{(B_{oi})_{i-1}} \right)$$

$$(S_g)_i = 1 - (S_o)_i - S_w$$

Step 5: Using the available relative permeability data, determine the relative permeability ratio k_{rg}/k_{ro} that corresponds to the gas saturation at P_i and compute the instantaneous GOR (R_i) at P_i as:

$$R_i = (R_{so})_i + \left(\frac{K_{rg}}{K_{ro}}\right)_i \left(\frac{\mu_o B_o}{\mu_g B_g}\right)_i$$

It should be noted that all the PVT data in the expression must be evaluated at the assumed reservoir pressure P_i .

Step 6: Calculate again the cumulative gas production $(G_p)_i$ at P_i given as

$$(G_p)_{GOR,i} = (G_p)_{i-1} + \left[\frac{R_{i-1} + R_i}{2}\right] [(N_p)_i - (N_p)_{i-1}]$$

In which R_{i-1} represents the instantaneous GOR at P_{i-1} . If P_{i-1} represents the initial reservoir pressure, then set $R_{i-1} = R_{si}$.

Step 7: The total gas produced $(G_p)_i$ during the first prediction period as calculated by the material balance equation $\{(G_p)_{MBE,i}\}$ is compared to the total gas produced as calculated by the GOR equation $\{(G_p)_{GOR,i}\}$. These two equations provide the two independent methods required for determining the total gas produced.

Therefore, if the cumulative gas production $\{(G_p)_{MBE,i}\}$ as calculated from Step 3 agrees with the value $\{(G_p)_{GOR,i}\}$ of Step 6, the assumed value of $(N_p)_i$ is correct and a new pressure may be selected and Steps 1 through 6 are repeated. Otherwise, assume another value of $(N_p)_i$ and repeat Steps 2 through 6.

Step 8: In order to simplify this iterative process, three values of $(N_p)_i$ can be assumed, which yield three different solutions of cumulative gas production for each of the equations (i.e., MBE and GOR equation). When the computed values of $(G_p)_i$ are plotted versus the assumed values of $(N_p)_i$, the resulting two curves (one representing results of Step 3 and the one representing Step 5) will intersect. This intersection indicates the cumulative oil and gas production that will satisfy both equations.

VII. SCHILTHUIS'S PREDICTION ALGORITHM⁶

Step 1: Assume value for the incremental oil recovery at the current pressure of interest given as:

$$\left(\frac{\Delta N_p}{N}\right)_1 \quad \& \quad \left(\frac{\Delta N_p}{N}\right)_2$$

Step 2: Determine the cumulative oil produced to the current pressure of interest by adding all the previous incremental oil produced.

$$\left(\frac{N_p}{N}\right)_1 = \sum \left(\frac{\Delta N_p}{N}\right)_1 \quad \& \quad \left(\frac{N_p}{N}\right)_2 = \sum \left(\frac{\Delta N_p}{N}\right)_2$$

Step 3: Determine the oil saturation from material balance equation given as:

$$S_o = (1 - S_{wi}) \left[1 - \frac{N_p}{N}\right] \left(\frac{B_o}{B_{oi}}\right)$$

The total fluid saturation can be calculated as:

$$S_L = S_o + S_w$$

The gas saturation is calculated as:

$$S_g = 1 - S_o - S_{wi}$$

Step 4: Determine the relative permeability ratio

$$\frac{k_{rg}}{k_{ro}} \text{ or } \frac{k_{rw}}{k_{ro}}$$

Step 5: Calculate the instantaneous gas-oil ratio at the current pressure of interest

$$R = R_{so} + \frac{K_{rg} \mu_o B_o}{K_{ro} \mu_g B_g}$$

Step 6: Calculate the average gas-oil ratio over the current pressure drop

$$R_{avg} = \frac{R_{i-1} + R_i}{2}$$

Step 7: Calculate the incremental gas production

$$\left(\frac{\Delta G_p}{N}\right)_1 = \left(\frac{\Delta N_p}{N}\right)_1 * R_{avg} \quad \& \quad \left(\frac{\Delta G_p}{N}\right)_2 = \left(\frac{\Delta N_p}{N}\right)_2 * R_{avg}$$

Step 8: Determine the cumulative gas produced to the current pressure of interest by adding all the previous incremental gas produced.

$$\left(\frac{G_p}{N}\right)_1 = \sum \left(\frac{\Delta G_p}{N}\right)_1 \quad \& \quad \left(\frac{G_p}{N}\right)_2 = \sum \left(\frac{\Delta G_p}{N}\right)_2$$

Step 9: Determine the cumulative produced gas-oil ratio given as:

$$(R_p)_1 = \frac{\left(\frac{G_p}{N}\right)_1}{\left(\frac{\Delta N_p}{N}\right)_1} \quad \& \quad (R_p)_2 = \frac{\left(\frac{G_p}{N}\right)_2}{\left(\frac{\Delta N_p}{N}\right)_2}$$

Step 10: Check for convergence

$$f(x_i) = \frac{\left(\frac{N_p}{N}\right)_{x_i} [B_o + ((R_p)_1 - R_s) B_g]}{(B_o - B_{oi}) + (R_{si} - R_s) B_g} - 1 = \text{Tolerance}$$

$$f(x_{i-1}) = \frac{\left(\frac{N_p}{N}\right)_{x_{i-1}} [B_o + ((R_p)_2 - R_s) B_g]}{(B_o - B_{oi}) + (R_{si} - R_s) B_g} - 1 = \text{Tolerance}$$

Step 11: If convergence is satisfied, then stop the iteration process, else calculate the new incremental oil recovery using the equation below and repeat the entire process.

$$\left(\frac{N_p}{N}\right)_{x_{i+1}} = \left(\frac{N_p}{N}\right)_{x_i} - \left[\frac{f(x_i) \left\{ \left(\frac{N_p}{N}\right)_{x_i} - \left(\frac{N_p}{N}\right)_{x_{i-1}} \right\}}{f(x_i) - f(x_{i-1})} \right]$$

Input Data of XY Field

Table 1: Reservoir properties of XY reservoir

Initial pressure	3200 psia
Bubble point pressure	3200 psia
Reservoir temperature	220 ^o F
STOIP	9655344 stb
Initial water saturation	23%

Table 2: PVT data of XY reservoir

Pressure (psia)	B _o (bbl/STB)	R _{so} (SCF/STB)	B _g (bbl/SCF)	Oil vis (cp)	Gas vis (cp)
3200	1.3859	1180	0.001383	0.84239	0.0238
2870	1.3784	1120	0.001618	0.89239	0.0233
2510	1.3603	1030	0.00184	0.9316	0.0231

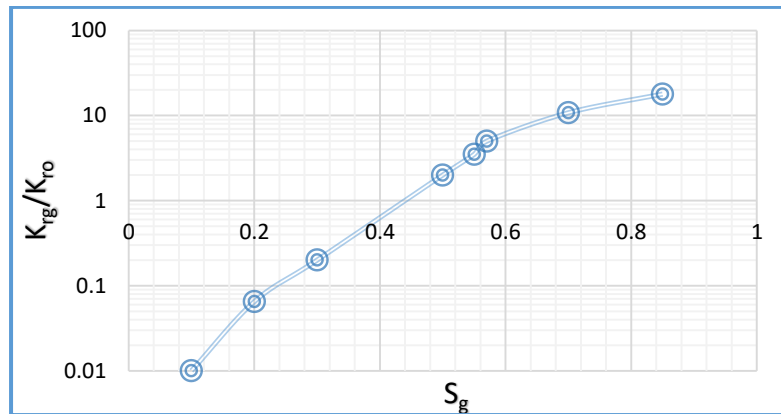


Figure 1: Relative permeability curve of XY reservoir

Result

The results in Tables 3, 4 and 5 respectively were obtained after several iterations with the algorithms stated above.

Table 3: Result of Turner's method

Assume cumulative oil production (Np)	531043.92 (stb)
cumulative gas production from MBE (Gp,MBE)	677542904.9 scf
oil saturation (So)	0.72371
gas saturation (Sg)	0.04628
relative permeability ratio (kr/kro)	0.01
instantaneous GOR	1445.8809 scf/stb
cumulative gas production from GOR (Gp,GOR)	697229066.5687 scf
absolute relative error (Ei)	2.93%

Table 4: Result of Tracy method

Assume GOR	1447 scf/stb
Estimated average instantaneous GOR	1313.5 scf/stb
Incremental cumulative oil production	511909.5183 stb
Oil saturation (So)	0.7252
Gas saturation (Sg)	0.0448
relative permeability ratio (Krg/Kro)	0.01
New instantaneous GOR	1446.284 scf/stb
Cumulative gas production	671650330.9 scf

Table 5: Result of Schilthuis method

Assume incremental oil recovery per STOIP	0.0529
cumulative oil produced per STOIP	0.0529
Oil saturation	0.7253
gas saturation	0.0447

relative permeability ratio	0.01
Instantaneous GOR	1446.284 scf/stb
Average GOR	1313.5 scf/stb
cumulative gas produced per STOIIP	69.4652
cumulative produced GOR	1313.142
Convergence error	-0.00146
cumulative oil produced (Np)	510767.69 stb
cumulative gas produced (Gp)	670710402 scf

The cumulative oil and gas production at 2870 psia for the three methods of prediction in Table 6 were obtained after several iterations with a convergence criteria of absolute relative error less than 5%.

Table 6: Summary of results

Parameter	Tarner's Method	Tracy's Method	Schilthuis' Method
Cum Oil Production (STB)	531043.9	511343.99	510767.69
Cum Gas Production (SCF)	6.9734×10^8	6.7165×10^8	6.7071×10^8

VIII. CONCLUSION

When production data are available, material balance techniques becomes an important tool for estimating reserves but predictions with these methods are not as effective as three dimension reservoir simulation techniques but can be used by engineers to get a feel of the reservoir. Hence, XY field prediction performance was carried out by comparing the result of cumulative oil and gas production from Tarner's, Tracy's and Schilthuis' methods after performing several iterations in Microsoft Excel as shown in Table 6. The results obtained with Tracy's and Schilthuis' methods indicate a closeness with the XY field performance prediction at 2870 psia, which is an indication that either Tracy's or Schilthuis method can be used to predict the XY field reservoir performance at lower pressures as the reservoir declines.

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APPENDIX

Manual Computation of Tarner's Method

Step 1: The pressure of interest = 2870 psia

Step 2: Assume the cumulative oil production $\{(N_p)_{2870}\}$ at 2870 psia = 531043.9 STB (i.e 5.5% of STOIIP).

Step 3: Calculate the cumulative gas production $(G_p)_{2870}$ by rearranging the MBE to give:

$$(G_p)_{MBE,2870} = 9655344 * \left(\{1180 - 1120\} - \left\{ \frac{1.3859 - 1.3784}{0.001618} \right\} \right) - 531043.9 \left\{ \frac{1.3784}{0.001618} - 1120 \right\}_{2870}$$

$$= 6.7693 \times 10^8 \text{ SCF}$$

Step 4: Calculate the oil and gas saturations at 2870 psia

$$(S_o)_{2870} = (1 - 0.23) \left[1 - \frac{531043.9}{9655344} \right] \left(\frac{1.3784}{1.3859} \right) = 0.7237$$

$$(S_g)_{2870} = 1 - (S_o)_i - S_w = 1 - 0.7582 - 0.23 = 0.0463$$

Step 5: Using the available relative permeability plot, the relative permeability ratio k_{rg}/k_{ro} that corresponds to the gas saturation $(S_g)_{2870}$

$$\left(\frac{K_{rg}}{K_{ro}} \right)_{2870} = 0.01 \text{ from Figure 1}$$

Compute the instantaneous GOR (R_i) at 2870 psia as:

$$R_{2870} = 1120 + 0.01 \left(\frac{0.89239 * 1.3784}{0.0233 * 0.001618} \right) = 1446.284 \text{ scf/STB}$$

Step 6: Calculate again the cumulative gas production at 2870 psia given as

$$(G_p)_{GOR,2870} = 0 + \left[\frac{1180 + 1446.284}{2} \right] [531043.9 - 0] = 6.9734 \times 10^8 \text{ SCF}$$

Step 7: Since the cumulative gas production are close, the absolute relative error is calculated as

$$|Error| = \left| \frac{6.9734 \times 10^8 - 6.7693 \times 10^8}{6.9734 \times 10^8} \right| \times 100\% = 2.9268\%$$

Manual Computation of Tracy Method

Step 1: The average reservoir pressure of interest = 2870 psia

Step 2: Calculate the values of the PVT functions ϕ_o , ϕ_g & ϕ_w where

The is no gas cap, hence $m = 0$

$$\phi_o = \frac{1.3784 - \{1120 * 0.001618\}}{(1.3784 - 1.3859) + \{(1180 - 1120) * 0.001618\}} = -4.84215$$

$$\phi_g = \frac{0.001772}{(1.3859 - 1.3784) + \{(1180 - 1120) * 0.001618\}} = 0.018062$$

Step 3: Assume $R_{assume} = 1447 \text{ SCF/STB}$ at 2870 psia

Step 4: Estimate the average instantaneous GOR (R_{avg}) at 2870 psia

$$R_{avg} = \frac{R_{i-1} + R_i}{2} = \frac{1180 + 1447}{2} = 1313.5$$

Step 5: Calculate the incremental cumulative oil production ΔN_p as:

Note

$$(N_p)_{i-1} = (G_p)_{i-1} = 0$$

$$\Delta N_p = \frac{9655344 - (0 * -4.84215) - (0 * 0.018062)}{-4.84215 + (0.018062 * 1313.5)} = 511343.99 \text{ STB}$$

Step 6: Calculate total or cumulative oil production from

$$(N_p)_{2870} = (N_p)_{i-1} + \Delta N_p = 0 + 511343.99 = 511343.99 \text{ STB}$$

Step 7: Calculate the oil and gas saturations at 2870 psia

$$S_o = \left[1 - \frac{511343.99}{9655344} \right] \left[\frac{1.3784}{1.3859} \right] [1 - 0.23] = 0.7253$$

$$S_g = 1 - 0.7253 - 0.23 = 0.0447$$

Step 8: Obtain the relative permeability ratio k_{rg}/k_{ro} at 2870 psia.

From the relative permeability curve given in Figure 1,

$$\frac{k_{rg}}{k_{ro}} = 0.010$$

Step 9: Calculate the new instantaneous GOR at time, i given as

$$R_{2870} = 1120 + 0.01 \left(\frac{0.89239 * 1.3784}{0.0233 * 0.001618} \right) = 1446.284 \text{ scf/STB}$$

Step 10: Compare the assumed GOR in Step 3 with the calculated GOR in Step 9.

$$R_{assume} = 1447 \text{ SCF/STB}$$

$$R_{2870} = 1446.28$$

Since these values are closed, thus the cumulative oil production is:

$$(N_p)_{3335} = 511343.99 \text{ STB}$$

Step 12: Calculate the cumulative gas production.

$$(G_p)_{2870} = 0 + 511343.99 * 1313.5 = 671650330.9 \text{ SCF}$$

Manual Computation of Schilthuis Method

Step 1: New guess

$$\frac{\Delta N_p}{N} = 0.0529$$

Step 2: The cumulative oil produced to the current pressure

$$\frac{N_p}{N} = \sum \frac{\Delta N_p}{N} = 0.0529$$

Step 3: Oil saturation from material balance equation given as:

$$S_o = (1 - 0.23)[1 - 0.0529] \left(\frac{1.3784}{1.3859} \right) = 0.7253$$

The gas saturation is calculated as:

$$S_g = 1 - 0.7253 - 0.23 = 0.0447$$

Step 4: The relative permeability ratio

$$\frac{k_{rg}}{k_{ro}} @ S_g = 0.01$$

Step 5: Instantaneous gas-oil ratio at the current pressure of interest

$$R = 1120 + 0.01 \left(\frac{0.89239 * 1.3784}{0.0233 * 0.001618} \right) = 1446.284 \text{ scf/STB}$$

Step 6: The average gas-oil ratio over the current pressure drop

$$R_{avg} = \frac{R_{i-1} + R_i}{2} = \frac{1180 + 1446.284}{2} = 1313.142 \text{ scf/STB}$$

Step 7: Calculate the incremental gas production

$$\frac{\Delta G_p}{N} = \frac{\Delta N_p}{N} * R_{avg} = 0.0529 * 1313.142 = 69.4652$$

Step 8: The cumulative gas produced to the current pressure of interest

$$\frac{G_p}{N} = \sum \frac{\Delta G_p}{N} = 69.4652$$

Step 9: The cumulative produced gas-oil ratio given as:

$$R_p = \frac{G_p}{\frac{\Delta N_p}{N}} = \frac{69.4652}{0.0529} = 1313.142$$

Step 10: Check for convergence

$$Tolerance = \pm 2\%$$

$$f\left(\frac{N_p}{N}\right) = \frac{N_p}{N} \left[\frac{B_o + ((R_p)_1 - R_s) B_g}{(B_o - B_{oi}) + (R_{si} - R_s) B_g} \right] - 1 = Tolerance$$

$$f\left(\frac{N_p}{N}\right) = \frac{0.0529[1.3784 + (1313.142 - 1120)0.001618]}{(1.3784 - 1.3859) + (1180 - 1120)0.001618} - 1 = -0.00146 < \pm 2\%$$

Step 11: If convergence is satisfied, thus the iteration process is stopped.

Therefore, the incremental oil recovery at 2870 psia is 0.0529. Given the STOIP = 9655344. It implies that the cumulative oil produced at 2870 psia is:

$$N_p = 0.0529 * N = 0.0529 * 9655344 = 510767.69 \text{ STB}$$

The cumulative gas produced at 2870 psia:

$$\frac{G_p}{N} = 69.4652$$

$$G_p = 69.4652 * N = 69.4652 * 9655344 = 670710402 \text{ SCF} = 6.7071 \times 10^8 \text{ SCF}$$

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